A modal age at death approach to forecasting mortality

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Abstract

Recent studies have shown that there are some advantages in forecasting mortality with other indicators than death rates. In particular, the age-at-death distribution provides readily available information on central longevity measures: mean, median and mode, as well as information on lifespan variation. The modal age at death has been increasing linearly since the second half of the 20th century, providing a strong basis to extrapolate past trends. We develop a model to forecast the age-at-death distribution that directly forecasts the modal age at death and lifespan variation while accounting for dependence between ages. We forecast mortality at age 40 and above in six Western European countries. The introduced model increases forecast accuracy compared with other forecasting models and provides consistent trends in life expectancy and lifespan variation at age 40 over time.

Keywords: Mortality, Forecast, Modal age at death, Lifespan variation

1. Introduction

The rise in life expectancy over the last two centuries is one of the most remarkable achievements of human populations. Life expectancy was around 40 years old in the middle of the 19th century and reached 87 years for Japanese females in 2020 (Oeppen and Vaupel 2002; HMD 2022). This constant increase has led to important demographic and societal changes, such as population growth and aging. Due to these continuous mortality changes and related consequences, public and private institutions rely on mortality forecasting to anticipate, for instance, healthcare and pension costs. The last few decades have witnessed an important increase in the number of forecasting models.

One recurrently used model to forecast mortality is the Lee-Carter (LC) model (Lee and Carter 1992), which forecasts age-specific death rates log bilinearly. The advantages of the LC model include its simplicity, limited subjective judgement and direct forecast of the risk of deaths over time. However, the accuracy of the model is often questioned. The LC model assumes a constant rate of mortality improvement in age-specific

death rates over time, but evidence shows that there has been accelerated mortality decline at older ages for some populations (Rau et al. 2008; Vaupel et al. 2021). The former assumption often leads to the method under-predicting life expectancy (Bergeron-Boucher and Kjærgaard 2022; Lee and Miller 2001). Variants of the LC model have been suggested over the years to improve the accuracy of the original model (Booth et al. 2006; Booth et al. 2002; Li and Lee 2005; Li et al. 2013).

Many forecasting models are based on the extrapolation of age-specific death rates, as they are indicative of the change in the risk of dying over time and, in addition, serve as the point of entry of the life table (Basellini and Camarda 2019). Mortality forecasts can have a number of other meaningful demographic indicators as input (Bergeron-Boucher et al. 2019a). Some authors suggest forecasting the age-specific death probabilities (Cairns et al. 2006; King and Soneji 2011). Using age-specific death rates or probabilities as input to the same model generally leads to similar forecasting trends (Bergeron-Boucher et al. 2019a). Other input indicators, such as life expectancy (Pascariu et al. 2018; Raftery et al. 2012; Torri and Vaupel 2012) and the age-at-death death distribution (Basellini and Camarda 2019; Bergeron-Boucher et al. 2017; Oeppen 2008), have gained popularity as they allow for changes in age-specific rates of mortality improvement (ASRMI).

Using life expectancy has the advantage of directly forecasting the average duration of life, and it is the most popular measure of longevity. Life expectancy, as an aggregate indicator, is less volatile than age-specific measures of mortality. As a result, the associated forecasting models are not only more robust but also more parsimonious (Bergeron-Boucher et al. 2019a). However, life expectancy does not provide any information about age-specific mortality trends and levels, and one must rely on an additional model to derive age-specific mortality from the life expectancy values (Pascariu et al. 2020; Ševčíková et al. 2016).

The age-at-death distribution readily provides information on central longevity measures: the mean, the median and the mode (Canudas-Romo 2010). While the mean (life expectancy) is the most used measure, the mode is increasingly seen as an alternative measure of longevity (Canudas-Romo 2008; Horiuchi et al. 2013). The modal age at death, M, is the age at which most adult deaths occur. It has been increasing in low-mortality countries since the 1930s–40s for females and since the 1970s for males (Bergeron-Boucher et al. 2015; Canudas-Romo 2010; Diaconu et al. 2016). An increase in the modal age at death indicates that the age-at-death distribution is shifting towards older ages, a dynamic interpreted as *postponement* of the mortality schedule. The latter, also referred to as *shifting mortality*, has been the dominant determinant of life expectancy increase since the middle of the 21st century (Bergeron-Boucher et al. 2015; Bongaarts 2009; Canudas-Romo 2008). Increases in the modal age at death are primarily driven by a decrease in mortality at older ages (Diaconu et al. 2020; Horiuchi et al. 2013), especially at ages above the mode (Canudas-Romo 2010), making this indicator particularly relevant for studying longevity extension.

Additionally, the age-at-death distribution provides information about the variability of lifetimes. The latter, also called lifespan variation, captures inequalities in lifespans from a population perspective. Lifespan variation can be directly calculated from the age-at-death distribution and is increasingly seen as a relevant

complement to life expectancy and the modal age at death (Tuljapurkar 2001; van Raalte et al. 2018; Vaupel et al. 2011). Lifespan variation has been decreasing in most low-mortality populations since the late 19th century (Edwards and Tuljapurkar 2005), which has led to a compression of mortality around the mode. Unlike *M*, mortality compression (or reduction in lifespan variation) is driven by a reduction in mortality at younger ages (Aburto et al. 2020). There is, generally, a negative correlation between life expectancy and lifespan variation, as both measures are sensitive to mortality changes at young ages. However, this is not a mechanical relationship, and many exceptions have been documented where lifespan variation increases with life expectancy, suggesting that both are distinct metrics (Aburto and van Raalte 2018; Brønnum-Hansen 2017; van Raalte et al. 2011; van Raalte et al. 2018).

Basellini and Camarda (2019) introduced an innovative model to directly forecast the location (mode) and variability of the age-at-death distribution based on a segmented transformation of the age-at-death distribution (STAD). The authors model and forecast the difference between a standard and an observed distribution, using a transformation function that depends on the change in the differences of the mode, differences in variability before the mode and differences in variability after the mode. This model directly captures the two main mortality dynamics: shifting and compression of mortality. Oeppen (2008) also developed a forecasting model based on the age-at-death distribution. He used compositional data analysis (CoDA) to model and forecast a redistribution of deaths across age-groups (usually from younger to older ages). Compositions are vectors containing positive values, representing parts of a whole, carrying relative information summing up to a constant, such as proportions. CoDA is a set of tools that allows for the correct modelling of compositions, including distributions (Aitchison 1982). The use of age-at-death distribution and CoDA in forecasting accounts for age-dependency: due to the sum constraints, deaths in the life table are directly dependent on each other on the aggregate level, such that the decrease in deaths at one age will lead to an increase in deaths in at least one other age-group. This property of the model resolved independence problems between mortality components in forecasting, including causes of death (Kjærgaard et al. 2019).

While the STAD model makes use of the regularities in the modal age at death, it does not account for the age-dependency as permitted with the CoDA model. In this paper, we develop a novel approach to forecast age-at-death distribution that directly forecasts the modal age at death and lifespan variation while accounting for the dependence between ages. We use a CoDA approach to forecast the distribution centered around the modal age at death and forecast *M* independently. We called our model the Mode model. Our model captures both the shifting and compression of mortality, directly modelling the two main mortality dynamics as two distinct metrics.

The paper is organised as follows. In Section 2, we describe the dataset used and the populations analysed. Section 3 presents the methodology. In Section 3.1, we describe how we estimate and forecast the modal age at death. In Section 3.2, we introduce a CoDA model to forecast the age-at-death distribution centred around the modal age at death. This model captures changes in lifespan variation. Sections 3.3 and 3.4 present

how we calculate prediction intervals and how we estimate forecasting accuracy via an out-of-sample analysis, respectively. In Section 4, we provide an illustration of the methods by forecasting mortality for both females and males in six Western European countries. Section 4.1 presents the parameters of the model and their interpretation; Section 4.2 presents the results of the out-of-sample analysis, comparing the introduced model with three other models; and Section 4.3 shows the forecast until 2050. In Section 5, we discuss the method and results, adding concluding remarks.

2. Data

We apply the method introduced in Section 3 to forecast mortality in six Western European countries: Denmark, France, the Netherlands, Portugal, Sweden and Switzerland. This selection provides a mixture of low, medium and high mortality levels, as well faster and slower rates of mortality progress. We use data from the Human Mortality Database (HMD 2022) for both sexes from 1960 to 2019 (the last year with available data for all countries), from age 40 to 110 by single year of age. Death counts and exposures are extracted from the HMD to calculate life tables. We use the life-table age-at-death distribution to forecast mortality.

3. Methods

3.1 Estimating and forecasting the modal age at death

The modal age at death (M) is the age at which most deaths occur. The estimation of M is not straightforward as the age-at-death distribution is not always smooth, and random fluctuations can result in multiple maxima of the density of adult deaths. Several approaches have been suggested to overcome the irregular patterns of deaths around the mode. Parametric models, like the Gompertz or Siler models, have been used to smooth the mortality curve and calculate the mode (Canudas-Romo 2008). However, these models assume specific mortality shapes. Non-parametric approaches have been suggested to estimate M, such as the Kannisto method (Kannisto 2001) or the P-Splines approach (Ouellette and Bourbeau 2011). In this paper, we follow the method suggested by Ouellette and Bourbeau (2011) to estimate M, where the age-at-death distribution is smoothed with a Poisson P-Spline approach (Eilers and Marx 1996), using the R package *MortalitySmooth* (Camarda 2012).

M has been increasing linearly since the middle of the 20th century in most developed countries (Horiuchi et al. 2013). As a result of this linear development, we forecast M with a random-walk with drift.

3.2 Forecasting the distribution around M

The age-at-death distribution around M can be expressed as a vector $\boldsymbol{D} = [d_{M-20}, d_{M-19}, \dots, d_{M-1}, d_M, d_{M+1}, \dots, d_{M+20}]$ with d_x being the life table deaths. This vector can be interpreted as an indicator of lifespan variation, that is how compressed the lifespan distribution around the mode is. The location (M) and shape ($\boldsymbol{D}$) of age-at-death distributions are then forecast separately, i.e., we

model and forecast the shifting and compression of mortality as two separate metrics. Figure 1 illustrates changes in *M* and **D** over time for French males.

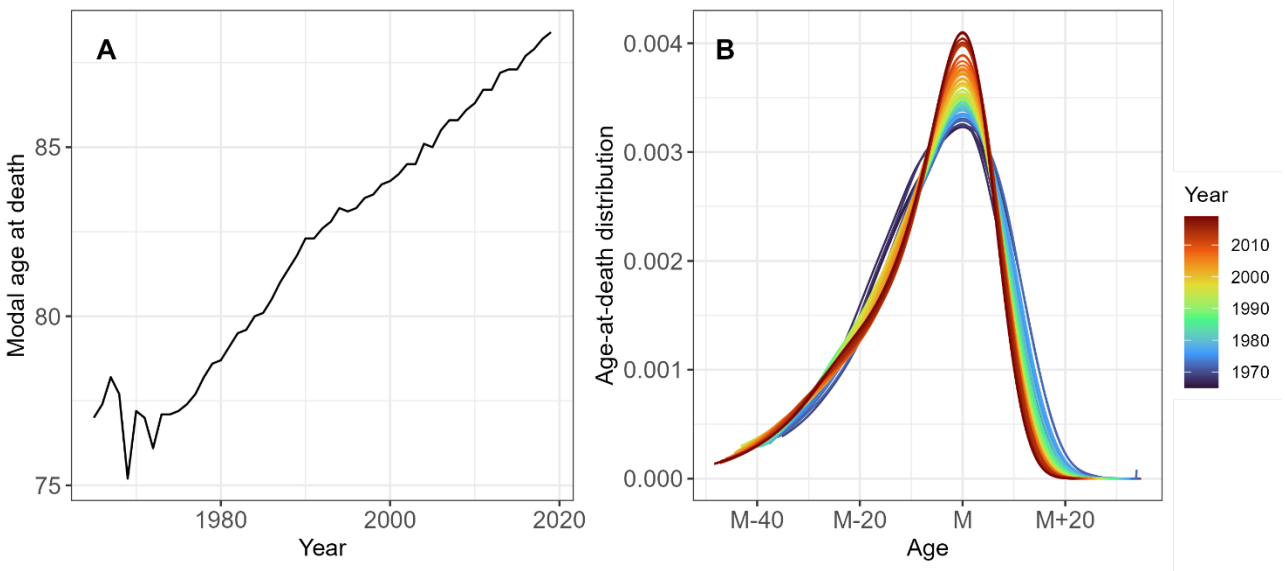

Figure 1. Modal age at death (A) and age-at-death distribution around the mode (B), French males, 1965-2019.

The smoothed age-at-death distribution around *M* can be forecast by the compositional data analysis approach of Oeppen (2008). In case of mortality compression, the model forecasts a redistribution of deaths towards *M* and is defined as

$$clr\left(d_{t,x-M} \ominus \alpha_{x-M}\right) = \kappa_t \beta_{x-M} + \varepsilon_{t,x-M}\,, \qquad (1)$$

where $d_{t,x-M}$ is the age-at-death distribution at time *t* and age *x*-*M*, while α_{x-M} is the age-specific average of $d_{t,x-M}$ over time. The *clr* is the centred log-ratio transformation and $\ominus$ is a perturbation operator (Aitchison 1982). The parameter κ_t is the time index, and β_{x-M} is the age-specific sensitivity to κ_t. The latter indicates which ages, relative to *M*, gain deaths over time and which lose deaths in relative terms. The parameters κ_t and β_{x-M} are estimated from a generalised singular value decomposition (GSVD). The GSVD allows giving weights to the age and time dimensions. We used a similar approach to that of Kjærgaard et al. (2019) to assign the weights. The age-specific weights are determined by the mean age-at-death distribution over time, thus giving more weight to the mode and the ages around it. The weights on the time dimension (ω_t) follow the approach of Hyndman et al. (2013), where more weight is given to the last years observed:

$$w_t = \rho(1-\rho)^{T-t}, \qquad (2)$$

where ρ determines the percentage weight on the most recent year (T). We used a ρ of 5% as suggested by Kjærgaard et al. (2019).

The parameter κ_t is not always linear. This is also observed when looking at measures of lifespan variation (Edwards and Tuljapurkar 2005; van Raalte et al. 2018). When forecasting non-linear trends, Hyndman and Athanasopoulos (2018) recommend using a natural cubic smoothing spline, which is a cubic spline with some constraints so that the spline function is linear at the end (Hyndman et al. 2012). We use this approach to forecast κ_t.

As M increases, the age-range that supports the distribution of deaths in the forecast varies over time. More precisely, we obtain less and less information about the left tail of the distribution and mortality at young ages. To remedy this problem, we assume that the missing values in the left tail of the distribution are equal to $d_{t,x+1} * \widehat{R_x}$, where $\widehat{R_x}$ is the $d_{t,x}$ ratio between two consecutive ages at the last year of observation, $\widehat{R_x} = \frac{d_{T,x}}{d_{T,x+1}}$. We also tested extrapolating the left tail using the penalised composite link model for ungrouping (Rizzi et al. 2015) and a monotonic interpolating spline. The model based on ratios provides the most satisfactory results. As $d_{t,x}$ values are usually small at the beginning of the selected age interval, how we estimate the missing $d_{t,x}$ has only minor impact on the forecasting results.

3.3 Prediction intervals

Prediction intervals are estimated by a bootstrapping procedure based on fitting errors from both models used to forecast M and κ_t. Forecasts are calculated for all combined simulations of M and κ_t, and 95% confidence intervals are calculated from the simulations by taking the 2.5 and 97.5 percentiles.

3.4 Out-of-sample approach

We use an out-of-sample approach based on different fitting periods and forecast horizons to evaluate forecasting accuracy. Forecasts are sensitive to both these components. Mortality for each country is forecast starting between 1994 and 2014 and until 2019, representing a forecast horizon varying from 25 to 5 years. For each forecast horizon, the fitting period starts in either 1960, 1965 or 1970 and has a length varying from 55 to 25 years. As a result, we make 63 forecasts for each country and sex.

Accuracy is measured by the root-mean-square error (RMSE), using different indicators: (1) Life expectancy at age 40, (2) the modal age at death, (3) lifespan variation and (4) the logarithmic age-specific death rates. Lifespan variation is measured by the average years of life lost at age 40, labelled $e^{\dagger}$ (Vaupel and Romo 2003). In addition, the accuracy of the prediction intervals is found by assessing the percentage of observed values falling within the intervals. The accuracy of the Mode model is compared with that of the LC, CoDA and STAD models.

4. Results

4.1 Parameters

Figure 2 shows the estimated parameters M, κ_t and β_{x-M} , as well as their forecast for French males and females. M has been increasing linearly since the 1970s in France for males and before the 1960s for females. Similar results are also found for the other five countries, with the beginning of the increase starting in the 1990s at the latest for Danish males. Thus, there is a postponement of the mortality schedule in the last three to six decades across all countries and sexes. Trends in the mode have generally been linear, which makes them easily extrapolatable by a linear model.

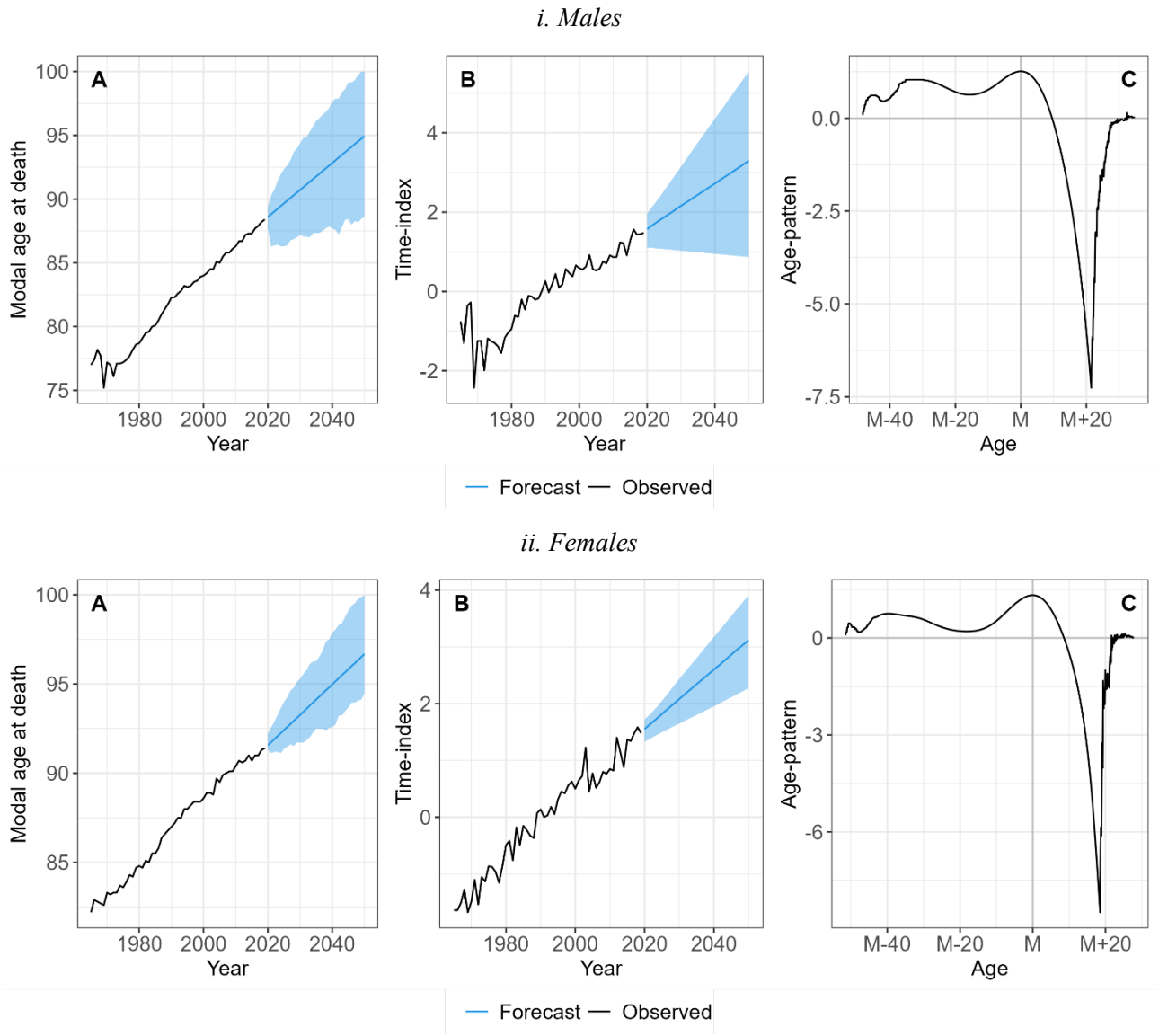

Figure 2. Observed and forecast model parameters for the (A) mode, (B) time-index and (C) age-pattern, French males (i) and females (ii), 1965-2050.

The parameters κ_t and β_{x-M} describe the change in the age-at-death distribution around *M* and capture how lifespan variation evolves over time. When κ_t increases, deaths are redistributed from ages with negative β_{x-M} towards ages with positive β_{x-M}. Figure 2.C shows that deaths become increasingly redistributed towards *M* and the ages around it over time, capturing a compression of mortality around *M*. The β_{x-M} profile is very similar across countries for males, but more variations are observed for females (see Appendix A). However, for all the selected countries and for both sexes, there was a redistribution of deaths from ages about 10 years and more above the mode towards the modal age at death (highest value) and ages around it.

4.2 Out-of-sample analysis

Tables 1 shows the mean errors in forecasting life expectancy at age 40 (e_{40}), the modal age at death (*M*), lifespan variation ($e^{\dagger}$) and the accuracy of the prediction intervals for e_{40} for four models: Mode, STAD, CoDA and LC. Accuracy levels vary across sexes, countries and models. However, on average, the models using the modal age at death as input (Mode and STAD) are more accurate in predicting *M* and e_{40}, for both males and females. As the mode has been the main driver of changes in life expectancy since the middle of the 20th century (Bergeron-Boucher et al. 2015), it should not be surprising that models which can best predict *M* can also better predict life expectancy.

Bohk-Ewald et al. (2017) note that the evaluation of forecasting models based on life expectancy alone is not sufficient as such an evaluation cannot determine whether or not the underlying mortality developments are plausible. The authors also suggest evaluating whether lifespan variation forecasts are plausible, so that the forecast can accurately predict the mortality shape. As shown in Table 1, forecast errors for $e^{\dagger}$ are generally smaller than those for *M* and e_{40} for all selected models. As a result, the accuracy of the models is not depreciated by looking at this indicator. While the STAD model has a fair forecast accuracy in predicting *M*, it provides a relatively low accuracy for $e^{\dagger}$. The Mode model was more accurate in forecasting $e^{\dagger}$, but the LC model was the model which provided, on average, the best accuracy for lifespan variation. However, the LC model is generally not accurate in forecasting the modal age at death, and consequently the life expectancy.

The CoDA model provides the most accurate prediction intervals, with 90.0% and 96.2% of the observed life expectancy values falling within the prediction's bands, for males and females, respectively. Using the Mode model yields 85.0% and 92.5% accuracy for males and females, respectively.

Table 2 shows the average forecast accuracy across countries based on age-specific death rates for broad age-groups. The Mode model was the most accurate in forecasting age-specific death rates below age 80. Mortality above age 80 was more accurately forecast by the CoDA model, followed by the Mode model. The CoDA model forecasts an increase in the ASRMI over time (Bergeron-Boucher et al. 2017), capturing more accurately the recent accelerating decline in mortality at older ages. In Appendix B, we show that the ASRMI at older ages also increase with the Mode model.

Table 1. Mean forecast errors across the 63 out-of-sample forecast for life expectancy at age 40, the modal age at death and lifespan variation ($e^{\dagger}$), as well as the prediction interval accuracy for life expectancy, forecast with the Mode, STAD, CoDA and LC models, for six countries and both sexes.

Males							
	Denmark	France	Netherlands	Portugal	Sweden	Switzerland	Mean (rank)
e_{40}							
Mode	1.66	0.42	1.49	0.68	**0.60**	**0.43**	**0.88 (1)**
STAD	**1.42**	1.66	**1.20**	**0.59**	0.79	1.09	1.13 (3)
CoDa	1.71	**0.39**	1.69	1.03	1.03	0.88	1.12 (2)
LC	1.58	1.00	1.38	0.95	1.17	0.93	1.17 (4)
95% PI							
Mode	75.5	**97.4**	56.1	**95.9**	88.1	**96.7**	85.0 (2)
STAD	64.7	68.7	**68.9**	94.0	84.0	75.1	75.9 (4)
CoDa	**80.6**	99.4	64.2	100.0	**98.1**	97.5	**90.0 (1)**
LC	40.7	98.2	62.9	94.0	91.0	89.2	79.3 (3)
M							
Mode	**2.29**	**0.28**	1.95	**0.98**	**0.70**	**0.44**	**0.91 (1)**
STAD	2.48	1.56	**1.75**	1.01	0.78	1.11	1.45 (2)
CoDa	2.59	0.47	2.44	1.42	1.28	1.09	1.55 (4)
LC	2.36	1.15	2.01	1.25	1.36	1.12	1.54 (3)
$e^{\dagger}$							
Mode	**0.17**	0.22	0.13	0.10	**0.11**	0.21	0.16 (2)
STAD	0.64	0.61	0.34	0.43	0.39	0.63	0.51 (4)
CoDa	0.34	0.11	0.17	0.08	0.19	0.17	0.18 (3)
LC	0.22	**0.11**	**0.08**	**0.08**	0.17	**0.15**	**0.13 (1)**
Females							
	Denmark	France	Netherlands	Portugal	Sweden	Switzerland	Mean (rank)
e_{40}							
Mode	1.05	0.53	**0.37**	**0.54**	0.25	**0.53**	**0.54 (1)**
STAD	**0.95**	0.98	0.41	0.83	0.41	0.74	0.72 (2)
CoDa	1.21	0.78	0.43	1.08	0.31	1.11	0.82 (3)
LC	1.26	**0.39**	0.42	1.58	**0.19**	1.21	0.84 (4)
95% PI							
Mode	81.6	86.7	**94.7**	99.8	96.9	**95.3**	92.5 (2)
STAD	68.3	**93.5**	95.3	**93.5**	94.7	95.4	90.1 (3)
CoDa	**97.0**	97.8	99.5	98.5	99.9	84.5	**96.2 (1)**
LC	55.7	86.1	81.6	83.9	**95.6**	78.8	80.3 (4)
M							
Mode	0.46	0.47	0.39	**0.52**	0.44	**0.71**	**0.50 (1)**
STAD	**0.43**	0.45	**0.36**	0.61	0.45	0.71	0.50 (2)
CoDa	0.62	0.34	0.42	1.12	0.23	0.99	0.62 (3)
LC	0.55	**0.29**	0.39	1.46	**0.19**	1.06	0.66 (4)
$e^{\dagger}$							
Mode	0.51	**0.10**	0.22	0.23	0.16	0.27	0.25 (2)
STAD	0.85	0.18	0.55	0.49	0.46	0.42	0.49 (4)
CoDa	0.53	0.33	**0.12**	**0.16**	**0.07**	0.28	0.25 (3)
LC	**0.48**	0.15	0.12	0.19	0.08	**0.24**	**0.21 (1)**

The Mode model offers the best accuracy trade-off across all compared indicators. On average, across countries the model ranks first in forecasting life expectancy at age 40, the modal age at death and age-specific death rates below age 80 and second in forecasting lifespan variation (first being the LC model), prediction intervals and mortality above age 80 (first being the CoDA model).

Table 2. Mean forecast errors for the age-specific death rates across six countries, fitting periods and forecast horizons, by broad age groups and all-ages; and number of countries with the lowest RMSE in brackets, by sex.

	Males			
	40-59	60-79	80-99	All ages
Mode	**0.113 (5)**	**0.114 (4)**	0.083 (3)	0.185 (0)
STAD	0.179 (0)	0.212 (0)	0.135 (0)	0.203 (0)
CoDa	0.151 (0)	0.148 (0)	**0.079 (3)**	**0.145 (3)**
LC	0.157 (1)	0.141 (2)	0.091 (0)	0.149 (3)
	Females			
	40-59	60-79	80-99	All ages
Mode	**0.096 (5)**	**0.079 (4)**	0.072 (2)	**0.117 (2)**
STAD	0.227 (0)	0.106 (1)	0.083 (1)	0.154 (0)
CoDa	0.151 (1)	0.127 (0)	**0.069 (0)**	0.125 (1)
LC	0.156 (0)	0.124 (1)	0.075 (3)	0.122 (3)

4.3 Forecasts

Figure 3 shows the life expectancy at age 40 and lifespan variation observed from 1965 to 2019 and forecasts until 2050 with the Mode and LC models for French males and females. The Mode model forecasts that life expectancy at age 40 in France will increase from 40.7 years in 2019 (between 39.4 to 42.4 years for the other countries) to 47.1 (44.5–48.9) years in 2050 for males and from 46.0 (43.6–46.0) years to 51.4 (47.9–51.4) years for females. In comparison, the LC model forecasts an increase to 45.1 years (between 43.2 and 46.7 years for the other countries) by 2050 for males and 50.1 (45.8–50.1) for females. Compared to the LC model, the Mode model forecasts a faster increase in life expectancy and the modal age at death but a slower decrease in lifespan variation. The LC model tends to produce a break in the trends at the jump-off year, particularly for the modal age at death. The LC generally assumes that future gains in life expectancy will result from faster compression and slower postponement than those in the past. As the increase in the mode is mainly due to mortality reduction above it, this result might come from the low and constant ASRMI at older ages assumed by the LC model. Meanwhile, the Mode model allows for increasing ASRMI at older ages. The model generally produces increasing ASRMI above age 85, constant or decreasing ASRMI between age 65 and 85, and mixed trends below age 65, depending on the country and sex (see Appendix B).

The forecasts with the Mode model generally continue the past trends in e_{40}, M and $e^{\dagger}$, without noticeable breaks in the trends. Figure 4 shows that the forecast trends in life expectancy at age 40 stay somewhat consistent among countries, but a divergence in trends is forecast across countries, with an increase in the range of life expectancy from 3.0 years in 2019 to 4.4 years in 2050 for males and from 2.4 to 3.5 years for females. A similar divergence is observed for M.

For females, M looks similar across all countries, except Portugal, until the early 1990s. Afterwards, the trends start to diverge, with a slower improvement for Denmark, the Netherlands and Sweden. The forecasts show persisting future divergence. For the Netherlands and Sweden, trends in e_{40} and M are consistent, which

might be an indication that mortality at older ages in these countries is not decreasing as rapidly as in France and Switzerland. For Danish females, while e_{40} stagnates between 1975 and 1995, we do not observe such stagnation in *M*, despite slower improvement after 1990. This result hints that the e_{40} stagnation for Denmark results from mortality worsening below the mode, as shown by the increase in $e^{\dagger}$ and by other studies (Christensen et al. 2010; Lindahl-Jacobsen et al. 2016).

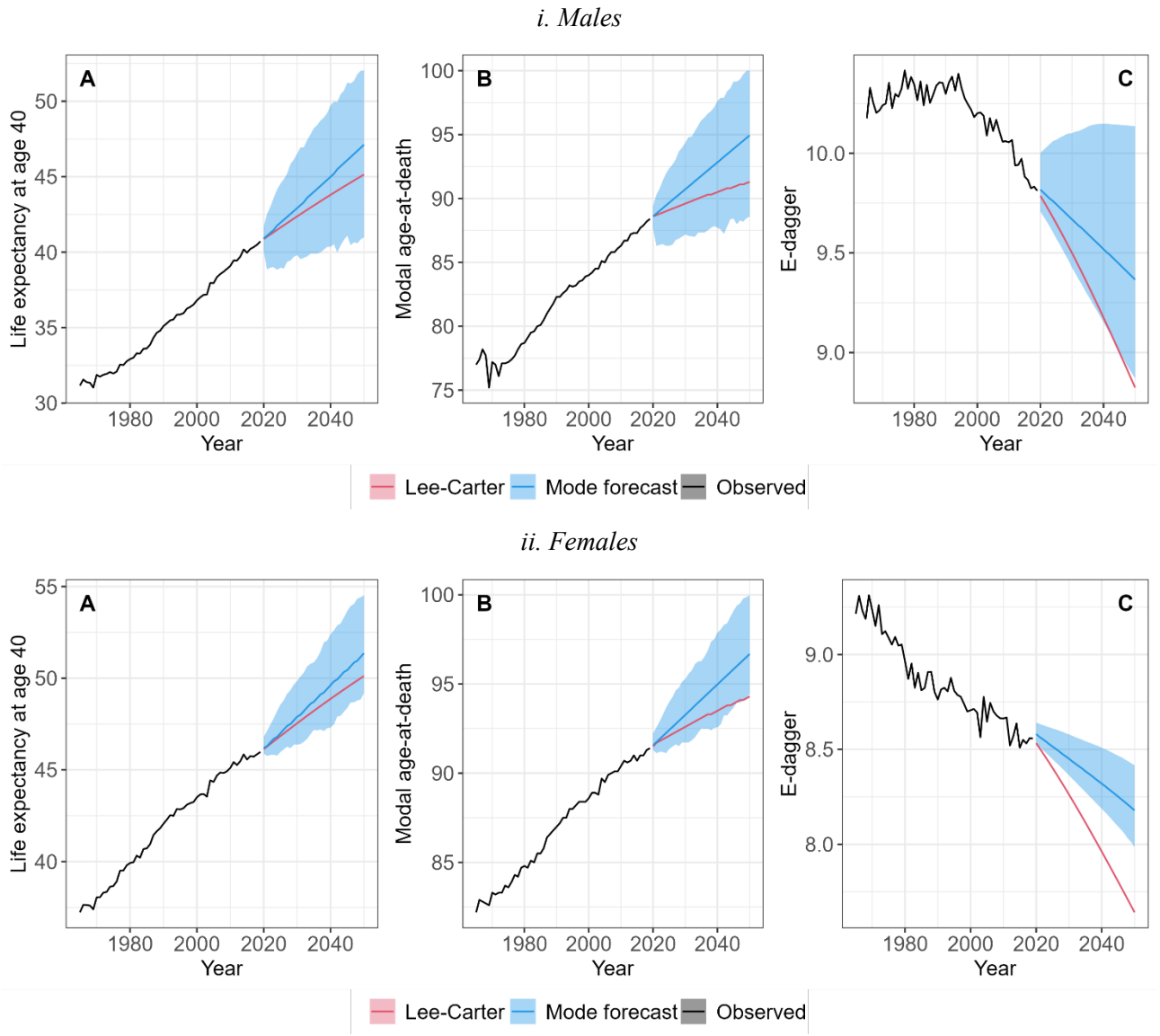

Figure 3. Life expectancy at age 40 (A), modal age at death (B) and lifespan variation (C) observed and forecast with the Mode and the Lee-Carter models, French males (i) and females (ii), 1965-2050.

For males, the divergence in e_{40} and *M* can be explained by the use of 1965–2019 as the fitting period for all countries. The mode only starts to increase in the 1980s for Dutch and Swedish males, and in the 1990s for Danish males, but it has been increasing since 1970 for France, Switzerland and Portugal. The forecast for

M is thus slowed down by fitting the model over a period of stagnation, which was more important in Denmark, the Netherlands and Sweden.

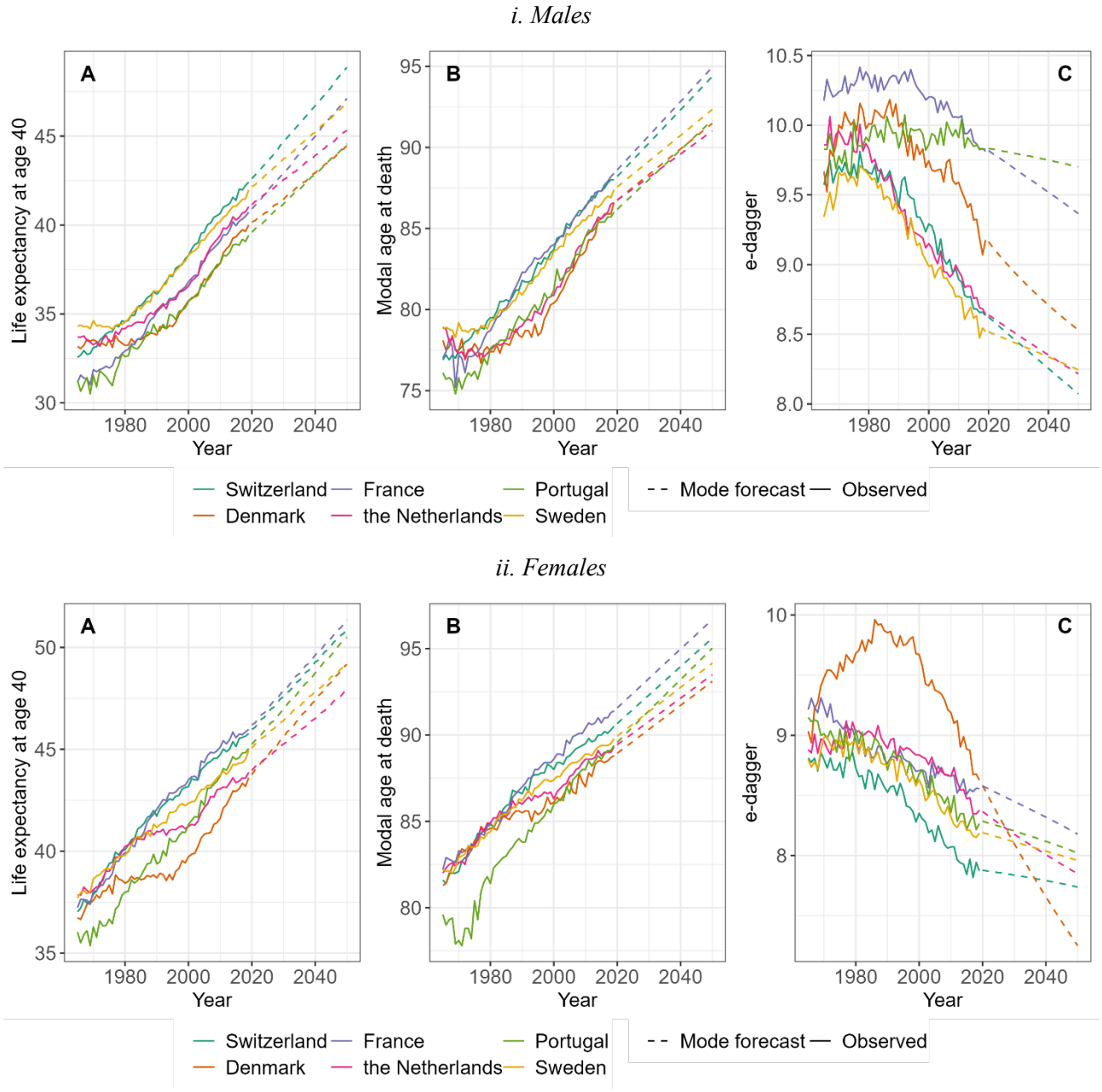

Figure 4. Life expectancy at age 40 (A), modal age at death (B) and lifespan variation (C) observed and forecast, for males (i) and females (ii) in Denmark, France, the Netherlands, Portugal, Sweden and Switzerland, 1965 to 2050.

Some divergence was also forecast for lifespan variation, mainly due to Portuguese males and Danish females. The range of values in $e^{\dagger}$ increased from 1.3 years to 1.6 years (1.3 without Portugal) for males and from 0.8 to 0.9 years for females (0.4 without Denmark). There have been small improvements in $e^{\dagger}$ for

Portuguese males and a rapid decrease for Danish females since the 1990s. The Mode model continues these trends in the forecast, which leads to a divergence between Denmark (females) and Portugal (males) and the other countries.

5. Discussion

Age-at-death distribution provides important information about mortality patterns and changes, as well as longevity (postponement) and lifespan variation (compression). In addition, using this indicator for forecasting can solve the dependency problems between components with the use of specific models and provide less biased forecasts than similar models based on death rates (Bergeron-Boucher et al. 2017; Kjærgaard et al. 2019). Yet, age-at-death distributions are rarely used as inputs for forecast models.

In this paper, we develop a model which builds on the advantages provided by the age-at-death distribution. The advantages include (1) the model uses the important regularity in the almost linear change in the modal age at death, and (2) accounts for changes in lifespan variation while (3) considering the dependence between ages. Other models, such as the CoDA and STAD models, have also made use of some of these advantages, but not fully. For example, the CoDA model builds on the third advantage, but not on the first two, while the STAD model builds on the two first advantages, but not on the third. Our analysis reveals that the Mode model could, on average, better predict both the modal age at death and life expectancy at age 40 in six Western European countries and for both sexes, compared with the other models. It also provides more accurate forecasts of the age-specific death rates and plausible trends in lifespan variation.

The modal age at death has increased linearly since the second half of the 20th century in many low-mortality populations (Horiuchi et al. 2013), providing a strong basis to extrapolate past trends. As opposed to life expectancy, the modal age at death is not sensitive to potential increases, or slowdowns, in mortality at younger ages. A stagnation or increase in life expectancy is observed over some periods of time in different countries, including Denmark and the United States. It is often caused by mortality worsening at young or middle-ages (Lindahl-Jacobsen et al. 2016; Woolf and Schoomaker 2019). For example, in Denmark, life expectancy stagnates for females between 1975 and 1995, mainly due to limited mortality improvement at middle-ages and an important burden from cancer mortality from specific birth cohorts (Bergeron-Boucher et al. 2019b; Christensen et al. 2010). However, no such stagnation is observed at older ages and, as a result, the modal age at death has been increasing in Denmark since the 1960s (or earlier) for females and the 1990s for males. The models that take advantage of this regular behaviour of the modal age at death (Mode and STAD) can produce more accurate mortality forecasts for the country. The modal age at death is usually better suited than the life expectancy to capturing the location of the age-at-death distribution, the speed of its shifting and the increase in longevity.

Measures of central tendencies, such as the mode or the mean, are not sufficient to determine whether mortality forecasts are plausible, as similar values of the mean or the mode can result from different mortality

developments (Bohk-Ewald et al. 2017). Lifespan variation provides useful information about mortality shapes and inequalities. In most populations, a reduction in lifespan variation is observed, with deaths becoming increasingly compressed around the modal age at death (Kannisto 2001). The Mode model directly models and forecasts this dynamic, with the β_{x-M} parameters capturing this redistribution of deaths towards the mode. The results show that the model can provide plausible lifespan variation trends and good forecasting accuracy.

Another advantage of the Mode model is its changing ASRMI over time, increasing at some ages and decreasing at others. Evidence shows that, in low-mortality countries, mortality decline is decelerating at younger ages but accelerating at older ages (Li et al. 2013; Rau et al. 2008; Vaupel et al. 2021). This pattern is referred to as a *rotation* (Li et al. 2013), which the Mode model is able to capture. The Mode and CoDA models allow for increasing ASRMI at older ages and generally produce more accurate forecasts of old-age mortality. However, for Swedish females, the ASRMI have stayed roughly constant in recent years, which might explain why the LC model performs better for this population (Bergeron-Boucher and Kjærgaard 2022). At younger ages, the ASRMI tend to stagnate or slow down. The Mode model is also able to account for this dynamic. For ages below the mode, assuming constant or decreasing ASRMI (implicit in the Mode and LC models) leads to better forecasts of the age-specific death rates.

Like other extrapolative models, the Mode model is sensitive to the fitting period selected. How to find the most relevant fitting period remains an open question. Generally, a longer fitting period provides more accurate forecasts, but it is also important to select a fitting period that reflects an ongoing or emerging dynamic (Bergeron-Boucher et al. 2019b; Janssen and Kunst 2007). The year when *M* starts to increase, capturing the mortality postponement dynamic, differs across countries and sexes. The use of the same fitting period across all studied populations, here used for consistency, affects the accuracy of the forecast. For males, using a more recent fitting period might have been more suitable.

Due to the ongoing COVID-19 pandemic, life expectancy has declined in 2020 and 2021 in most countries (Aburto et al. 2022). Extrapolative models, such as those tested in this paper, cannot account for mortality shocks. As the last year observed for all selected countries in the HMD was 2019, the predicted life expectancies for 2020 and 2021, and potentially 2022, will most likely be too high. Previous mortality shocks, such as the Spanish flu and the two World Wars, have led to a decrease in life expectancy for a short period of time, after which life expectancy has returned to its prior level within one or two years (Schoeley et al. 2022). In this context, the models tested would still be relevant to forecast post-pandemic mortality. However, whether or when mortality will return to its expected trajectory is still unclear.

The Mode model is limited to forecasting adult mortality patterns. Due to the shape of the human mortality pattern, using the full age range creates some fitting problems. There is a second mortality peak at birth. When estimating the age-at-death distribution centred around *M*, the peak of infant deaths will be allocated to ages further and further away from *M* over time, creating an artificially fast mortality decline at the relative ages where the peak used to be. A similar issue will arise if we consider the ages at which the

accident mortality hump appears, around ages 20 to 30. For this reason, we suggest limiting the forecast to adult-age mortality.

The introduced model has not been tested in cases where lifespan variation increases, such as in the United States since the 2010s (Acciai and Firebaugh 2019). It is unclear how the model will perform in such a context – this will depend on the β_{x-M} pattern. If β_{x-M} captures a redistribution of deaths towards ages below the mode, we might indefinitely forecast an increase in lifespan variation. In such a context, an extension of the model can be developed to allow for changes (rotations) in β_{x-M} over time, as previously suggested for the LC model (Li et al. 2013).

The model can also be extended to reflect other mortality processes. For example, a coherent version of the model can be developed to account for non-diverging trends among countries or between sexes by forecasting the differences between the modal age at death and a reference trend. The changes in lifespan variation can also be forecast between countries by applying the coherent CoDA model of Bergeron-Boucher et al. (2017). Causes-of-death information or smoking-related mortality can also be included in the model (Janssen et al. 2013; Kjærgaard et al. 2019). The use of compositional data analysis makes the Mode model particularly adept at forecasting mortality by cause, due to its component-dependence modelling. We forecast the modal age at death and the time-index of the age-at-death distribution centred around the mode as two separate trends because, sometimes, they behave inconsistently. However, the two measures are very often negatively correlated, and, in such cases, both indicators can be forecast dependently. Kjærgaard et al. (2019) suggest using a cointegrated vector error model to account for dependence between multiple time-indexes. It is, however, outside the scope of this paper to test all possible extensions of the introduced model.

The introduced Mode model can capture the two main mortality dynamics at adult ages: compression and shifting. These components are also well captured by the STAD model. The main difference between the two models is that the STAD model forecasts the difference between a standard age-at-death distribution and the observed distribution, while the Mode model directly forecasts the observed age-at-death distribution. The STAD model also forecasts three sets of parameters – the mode, as well as the variation before and after the mode – while the Mode model forecasts two sets of parameters – M and κ_t. An additional advantage of the Mode model is that it considers dependance between mortality components (ages), as does the CoDA model. Compared with the LC and CoDA models, both the Mode and STAD models improve forecasts of the mode, whose almost linear increase is the dominant dynamic in the last decades. The Mode model, however, provides a more accurate forecast of age-specific death rates and lifespan variation.

We use the modal age at death as a basis to forecast mortality as this indicator has been increasing with few or no breaks in many populations since the second half of the 20th century. This regularity, combined with M capturing mortality postponement and being easily combined with a measure of shape and inequalities, makes the use of the modal age at death appealing to forecast mortality. The introduced Mode model provides several advantages and can improve forecasting accuracy compared to other models. This is, however, only a

first step, and potential extensions of the model (e.g. its coherent extension) can help improve accuracy even more.

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Appendix

A. Comparison of age-patterns

Figure A1 shows the age-pattern (β_{x-M}) estimated with equation (1) in the main text across countries and for both sexes. There was a redistribution from death at ages 10 year above the mode towards the mode over time. Danish females also lost, in relative terms, deaths at above 30 years below the mode.

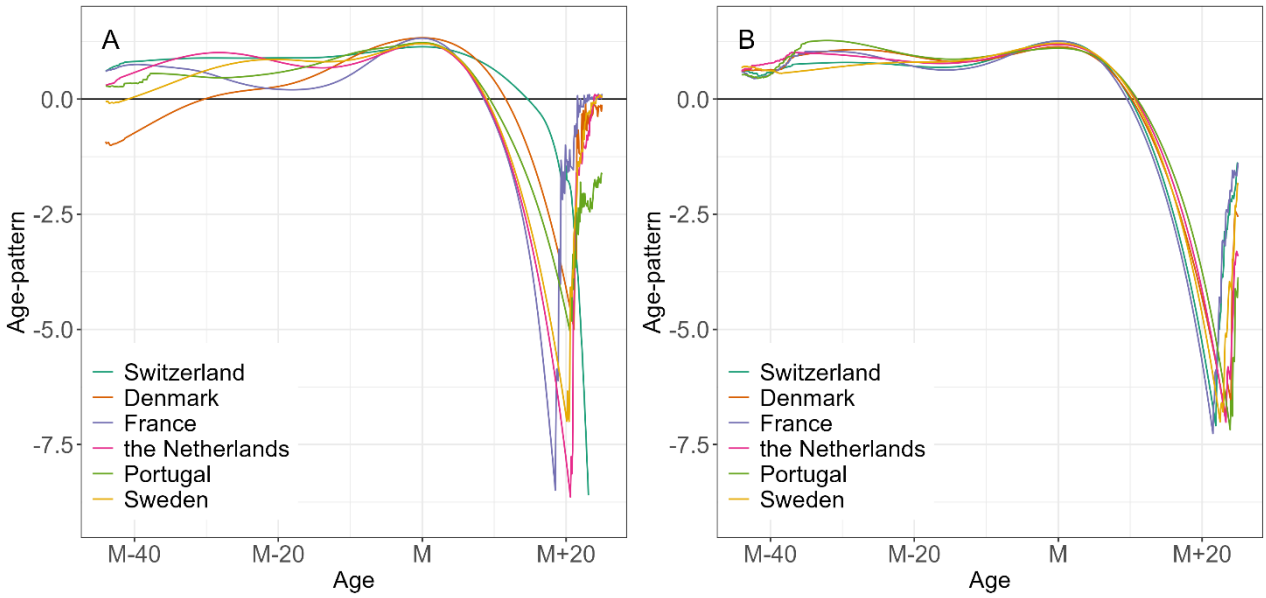

Figure A1. Age-pattern for the Mode model across countries and for females (A) and males (B).

B. Age-specific rates of mortality improvements (ASRMI)

Figure B1 shows the ASRMI for French males forecast with the Mode and LC models. Fluctuations are observed over time for the forecast ASRMI with the Mode model, so we smoothed the trends. The Mode model forecasts faster mortality improvements at most ages, compared to the LC model, except between ages 65 and 75. The former model also allows for variation over time in the ASRMI: sometimes the ASRMI increase over time (e.g. age 85–90), sometimes they decrease (e.g. age 75–80), remain constant (e.g. age 65–70) and sometimes there is an increase, followed by a decrease (e.g., age 50–55). Similar results are also found for French females (Figure B2) and for other countries (results not shown).

Figure B1. Age-specific rates of mortality improvements (ASRMI) forecast with the Mode model (full line) and the Lee-Carter model (dashed line) for French males, 2020 to 2050.

Figure B2. Age-specific rates of mortality improvements (ASRMI) forecast with the Mode model (full line) and the Lee-Carter model (dashed line) for French females, 2020 to 2050.